# AERODYNAMIC CHARACTERISTICS IDENTIFICATION: METHODS OVERVIEW AND APPLICATION OF ERROR EQUATION METHOD

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**Abstract.** The article gives overview of aircraft aerodynamic characteristics identification methods and their capabilities. There are concise descriptions of methods and their sorting according to different criterions. Case study results of error equation method application are presented at the second part. The data recording was simulated in Matlab Simulink. The simulation consists of input signal, equation of motion, generating of process and measurement noise and data recording. The process allows checking parameter estimation accuracy, because there are known unknown parameters. The whole procedure was simplified by limitation to 2D space. Airplane VUT 700 Specto has been chosen for this virtual experiment.

Keywords. Aerodynamic characteristics identification, Flight model simulation.

# **1** Introduction

Identification is a science which deals with description of some system. An identification procedure finds system layout model and its coefficients from input and output measurement. This paper is focused on identification of aerodynamic coefficients in the field of flight mechanics. The main reason for identification is to obtain mathematic flight model with all necessary coefficients. Other particular applications are for example investigation of flight performance and control and stability, analytical analyses validation or aerodynamic database creating for flight simulation.

# 2 Methods overview

Identification methods can be divided by some different criterions. Online and offline identification is one of them. Offline methods are applied after the measurement has been finished. All data are analyzed together. On the other side, online identification runs during the experiment. Data are analyzed subsequently. These methods bring immediate solution, have lower memory requirements but are less accurate. This paper deals with offline methods. The next division is into time and frequency domain. Frequency domain used to be preferred before PC processing, which got allow to calculate huge amount of measured data. Nowadays, frequency domain methods are used in helicopters identification. Time domain methods are mostly used in identification of fixed wing airplane. Undermentioned methods are in the time domain group.

#### 2.1 Error equation method

Error equation method (EEM) is very simple and simplicity is also the best advantage of the method. It is based on least square parameters estimation. The main disadvantage is neglecting of independent variables error. This disadvantage can be eliminated by very precise gages. Basic equation (1) minimizes a value of error square.

$$J(\theta) = \frac{1}{2} \sum_{k=1}^{N} \varepsilon^2(k) = \frac{1}{2} [Y^T - \theta^T X^T] [Y - X\theta]$$
<sup>(1)</sup>

*Y* is a matrix of dependent variables, *X* means a matrix of independent variables. It is supposed that the measured variables *Y* depend linearly on the independent variables *X* in each time k:

 $y_i(k) = \theta_1 x_1(k) + \theta_2 x_2(k) + \dots + \theta_n x_n(k) + \varepsilon(k)$ .  $\Theta$  is a matrix of unknown parameters and  $\varepsilon$  is error of dependent variables.

### 2.2 Output error method

Output error method (OEM) and EEM have some shared properties. They are very often used and both neglect a process noise. This means the measurement should be carried out in calm atmosphere without turbulence. Difference between these methods is in the parameter estimation. OEM uses Maximum likelihood (ML) estimation instead of least square estimation. The basic definition of ML estimation is in equation (2). There are N random independent observations  $(z_1, z_2, ..., z_N)$ . Likelihood function is defined:

$$p(z|\Theta) = \prod_{k=1}^{N} p(z_k|\Theta)$$
<sup>(2)</sup>

p is a probability that z occurs in the case of  $\Theta$ . The method finds parameters  $\Theta$  in a way that maximizes this probability. There is a limitation in this method. The input and output signals have to be independent. This assumption is easy to fulfil for stable aircraft. The numerical solution is iterative.

#### 2.3 Filter error method

This is more complex method which looks at process noise. This allows carrying out experiment in a gently turbulenced atmosphere. The method is better in convergence than OEM. The computation is stabilised by feedback and it allows analysing instable aircraft. The OEM and The Filter error method (FEM) have similar parameter estimation but differ in state estimator. It is not possible to compute dependent variables due to process noise in the FEM. Therefore the output *Y* is determined by a state estimator. The Kalman filter is used for linear systems and extended Kalman filter for nonlinear systems to estimate the state. There is FEM algorithm:

- 1)Starting values determination of  $\Theta$  and covariance matrix.
- 2)State estimation by Kalman filter.
- 3)Covariance matrix determination.
- 4)Optimization of parameters  $\Theta$ .
- 5) Iteration of 2, 3 and 4 until solution get converged.

### 2.4 Artificial neural networks

Artificial neural networks (ANN) should be divided into feedforward neural network and recurrent neural network. Both of them are used in aerodynamic characteristics identification. Unidirectional signal flow is characteristic for feedforward neural network. There is used behavioural model, it means we know model input and the output, but not inner processes. Output variables represent aerodynamic forces and moments coefficients. Recurrent neural network is characterized by following points. The

signal can flow through network in any direction. The unknown variables are outputs from single nodes. Recurrent neural network corresponds to state model or extended Kalman filter.

## **3** Case study of error equation method

#### 3.1 Flight simulation

Identification software is based on Lit.1 and its supporting materials. Matlab and Matlab simulink are used for specific application of virtual experiment. The measurement with real airplane is substituted by flight simulation to validate identification method and its accuracy. Model layout is demonstrated in the Fig. 1. Orange subsystems represent inputs, yellow simulation loop and blue means simulation outputs. The simulation is restricted on symmetric flight manoeuvre which is initialized by elevator deflection and thrust variation. Flight simulation has following progress. Firstly, input data set up is implemented. Initial flight regime conditions are solved and then flight simulation can be started. Data measurement and gust speed subsystems are provided with measurement and process noise implementation. Airplane VUT 700 Specto has been chosen for this experiment. It is small remote controlled airplane with span over 4 m and maximum take-off weight 20 kg.

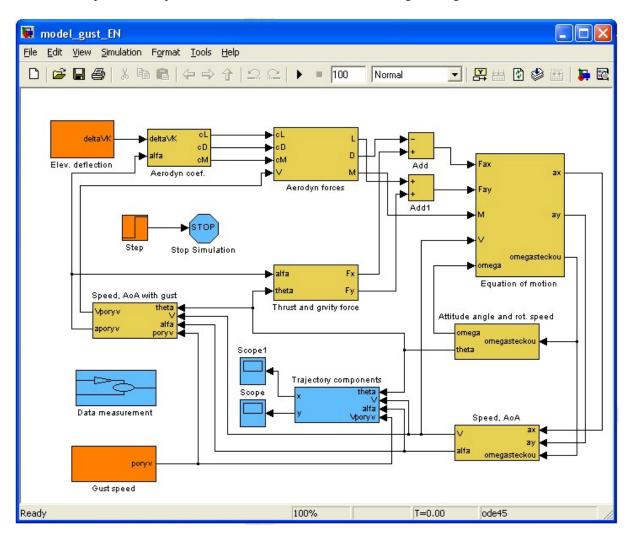


Figure 1: Flight simulation model layout.

### **3.2 EEM identification**

Following quantities were measured in the flight simulation: accelerations in x and y direction, angular rate, attitude angle, angle of attack, velocity, elevator deflection, thrust, temperature and static and dynamic pressure. A preprocessing is the next step after measured data loading. The measured data are converted into aerodynamic coefficients. Then there is carried out the least square parameter estimation. Computed and measured data are compared in graphs and also by R-square statistics. In the Fig. 2. there are compared identified coefficients with estimated coefficients in two cases. The first is without measurement noise and the second has low intensity of noise.

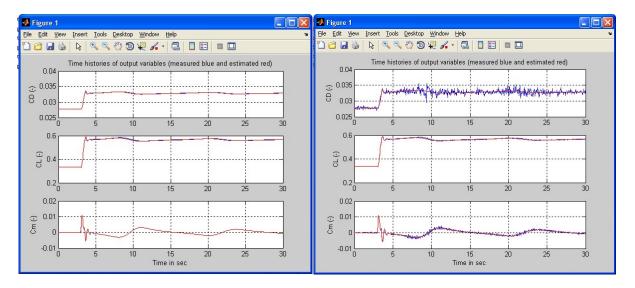


Figure 2: Measured and estimated parameters comparison.

There were estimated eight parameters from lift, drag and moment equations (3):

$$c_{L} = c_{L0} + c_{L}^{\alpha} \cdot \alpha + c_{L}^{\delta} \cdot \delta$$

$$c_{D} = c_{D0} + c_{D}^{cL} \cdot c_{L}^{2}$$

$$c_{mL} = c_{m0} + c_{m}^{\alpha} \cdot \alpha + c_{m}^{\delta} \cdot \delta$$
(3)

Parameters estimation was very precise in a condition without any noise. Estimated and measured parameters differ only in a few percent. This deviation is caused by numerical solution and by rounding. The results shows table 1.

Variables	cL0	cLα	cLδ	cD0	cDcL	Cm0	cmα	cmδ
Deviation [%]	2.00	1.42	0.28	0.05	2.24	0.03	0.04	0.06

Table 1: Measured and estimated parameters deviation.

Figure 3. shows influence of noise on some parameters solution accuracy. Measurement noise affection has exponential character and influences the results strongly. The noise was simulated as white noise with amplitude value up to ten percent of measured signal. The noise was applied on each measured variables. Reached parameters precision makes results useless for practical purposes. There are two possibilities how to decrease the inaccuracy. The measured data should be filtered before identification algorithm application. This analysis was carried out without filtering because data filter implemented inaccuracies in the case without any noise. The second possibility is to make more manoeuvres and more virtual experiments to reach better results.

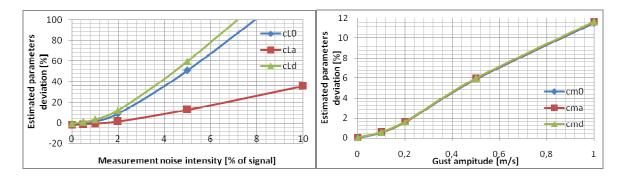


Figure 3: Estimated parameters deviation.

Process noise has been simulated as a gust with amplitude up to vertical velocity 1 m/s. The results affection has lower order in comparison with measurement noise. The reason can be explained through the simulation model. The gust changes angle of attack and it immediately influences aerodynamic coefficients. There is not any system delay. This reason shows necessity to enlarge the model by measurement delay to analyze the process noise.

# 4 Conclusion

The paper brings brief identification methods overview. The equation error method has been applied on the simulated measurement data. This virtual experiment cannot fully represent real measurement, but it is great help to debugging identification software. There are some necessary modifications to reach satisfactory results. It is some variables delay implementation, measurement noise correction, which responds to real measurement noise values and data filtering.

# References

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